Math II Exam Review
Variation A-CED.1

Direct Variation – A function that can be written in the form $y = kx$, where $k$ is the constant of variation.

Inverse Variation – A function that can be written in the form $y = \frac{k}{x}$, where $k$ is the constant of variation.

Joint Variation – A function that combines direct and inverse variations

For example:  
$y$ varies directly with the square of $x$  
$y = kx^2$

$y$ varies inversely with the cube of $x$  
$y = \frac{k}{x^3}$

$z$ varies jointly with $x$ and $y$  
$z = kxy$

$z$ varies jointly with $x$ and $y$ and inversely with $w$  
$z = \frac{kxy}{w}$

1. Suppose that $y$ varies inversely with the square of $x$, and $y = 50$ when $x = 4$. Find $y$ when $x = 5$.

\[
\begin{align*}
  y &= \frac{50}{x^2} \\
  50 &= \frac{k}{4^2} \\
  16 &= \frac{k}{50} \\
  k &= 800
\end{align*}
\]

2. Suppose that $y$ varies directly with $x$ and inversely with $z^2$, and $x = 48$ when $y = 8$ and $z = 3$. Find $x$ when $y = 12$ and $z = 2$.

\[
\begin{align*}
  y &= \frac{kx}{z^2} \\
  8 &= \frac{k(48)}{3^2} \\
  48 &= \frac{48k}{9} \\
  72 &= 48k \\
  k &= 1.5
\end{align*}
\]

\[
\begin{align*}
  y &= \frac{1.5x}{z^2} \\
  12 &= \frac{1.5x}{2^2} \\
  48 &= 1.5x \\
  x &= 32
\end{align*}
\]
3. A salesperson’s commission varies directly with sales. For $1000 in sales, the commission is $85. 

   \[ \text{commission} = K \cdot \text{Sales} \]

   \[ \frac{85}{1000} = k \frac{1000}{1000} \]

   \[ k = 0.085 \]

   \[ c = 0.085(2300) \]

   \[ c = 195.50 \]

4. The number of rotations of a bicycle wheel varies directly with the number of pedal strokes. Suppose that in the bicycle’s lowest gear, 6 pedal strokes move the cyclist about 357 inches. In the same gear, how many pedal strokes are needed to move 100 feet?

   \[ n = K \cdot p \]

   \[ \frac{357}{6} = 59.5 \]

   \[ K = 59.5 \]

   \[ p = 20.16 \]

   So about 20 strokes

5. If \( y \) varies directly with \( x \) and \( y \) is 18 when \( x \) is 6, which of the following represents the situation?

   A. \( y = 24x \)
   B. \( y = 3x \)
   C. \( y = 12x \)
   D. \( y = \frac{1}{3}x \)

   \[ y = Kx \]

   \[ 18 = K(6) \]

   \[ K = 3 \]

6. The number of bags of grass seed \( n \) needed to reseed a yard varies directly with the area \( a \) to be seeded and inversely with the weight \( w \) of a bag of seed. If it takes two 3-lb bags to seed an area of 3600 ft², how many 3-lb bags will seed 9000 ft²?

   A. 3 bags
   B. 4 bags
   C. 5 bags
   D. 6 bags

   \[ n = \frac{a}{w} \]

   \[ n = \frac{a}{600w} \]

   \[ n = \frac{900}{600} \]

   \[ n = 1.5 \]

7. The volume, \( V \), of a certain gas varies inversely with the amount of pressure, \( P \), placed on it. The volume of this gas is 175 cm³ when 3.2 kg/cm² of pressure is placed on it. What amount of pressure must be placed on 400 cm³ of this gas?

   A. 1.31 kg/cm²
   B. 1.40 kg/cm²
   C. 2.86 kg/cm²
   D. 7.31 kg/cm²

   \[ V = \frac{K}{P} \]

   \[ 175 = \frac{K}{3.2} \]

   \[ 400 = \frac{560}{P} \]

   \[ K = 560 \]

   \[ 100P = 560 \]

   \[ P = 5.6 \]
8. The amount of time it takes to build a road varies inversely with the number of workers building the road. Suppose it takes 50 workers 8 months to build the road.
   
   A. What is the constant of variation?
   
   \[ \frac{T}{w} = \frac{k}{50} \quad k = 400 \]

   B. Write an equation that could be used to determine how long it would take \( n \) workers to build the road. (Be sure to define the variables.)
   
   \[ \text{Time} = \frac{400}{\text{workers}} \quad T = \frac{400}{n} \quad T = \text{time} \quad n = \text{workers} \]

   C. How much faster would 60 workers build the road than 50 workers?
   
   \[ T = \frac{400}{60} \quad T = \frac{400}{50} \]
   
   \[ T = 6.7 \quad T = 8 \]
   
   It would take 1.3 less months to finish.

9. The time, \( t \), in hours that it takes \( x \) people to plant \( n \) trees varies directly with the number of trees, and inversely with the number of people. Suppose 6 people can plant 12 trees in 3 hours. How many people are needed to plant 28 trees in 5 hours and 15 minutes?

   A. 6    B. 7    C. 8    D. 9

   \[ T = \frac{K_1}{X} \quad T = \frac{1.5n}{x} \]
   
   \[ \frac{3}{6} = K \quad \frac{5.25}{x} = 1.5 \quad 5.25 \times x = 42 \]
   
   \[ x = 8 \]

10. The force, \( F \), acting on a charged object varies inversely to the square of its distance, \( r \), from another charged object. When the two objects are 0.64 meter apart, the force acting on them is 8.2 Newtons. \textit{Approximately} how much force would the object feel if it is at a distance of 0.77 meter from the other object?

   A. 1.7 Newtons
   B. 5.7 Newtons
   C. 11.9 Newtons
   D. 12.9 Newtons